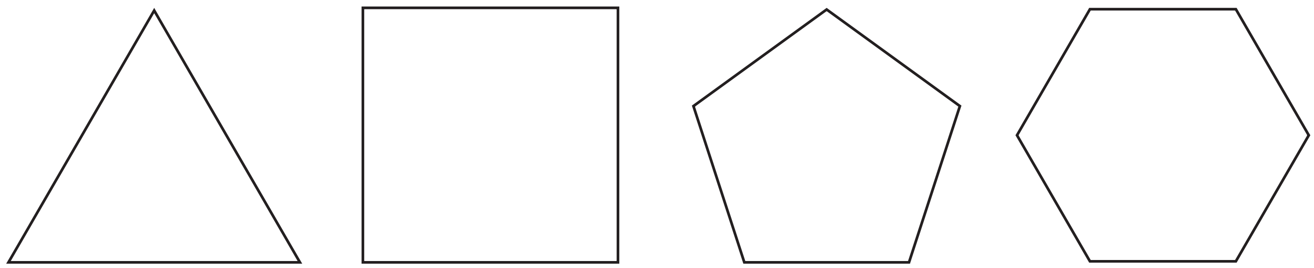
Consider, for example, what object comes next in the following sequence:



Or, for another example, what object comes next in the following sequence: *Monday, Tuesday, Wednesday, Thursday...*?

All of the sequences we will consider are "nice" in that the sequence can be extended in a logical manner. Mathematicians do consider lists that do not have a pattern or formula to be sequences (a sequence of random numbers, for example), but we will restrict ourselves to nice sequences that we can extend logically and predict.

Even within the realm of numerical sequences that can be extended, there are problems assuming that sequences are what they first appear to be. Any numerical sequence can, in theory, be extended in an infinite number of different ways that may or may not agree with the perceived rule or pattern for the sequence. For example, we said earlier that 2, 4, 6, 8, … appears to be a list of even numbers, and so extended it with 10, 12, and 14. However, this sequence does not have to be a list of even numbers, and there are many different ways to extend this sequence. The sequence 2, 4, 6, 8, 10, 58, 252, 734… is a valid mathematical sequence, although it is very different from how we expect a sequence beginning with 2, 4, 6, 8 to continue. In this resource guide, if a sequence seems to follow a simple pattern, we will generally assume it continues this pattern, but strictly speaking this is not necessarily the case.

In order to communicate effectively about sequences, mathematicians have developed some common notation.

NOTATION

The position of a term in a sequence is called the **index** of the term. The terms of a sequence are denoted by a variable (usually a or x) and an index, with the index written as a subscript. Unless otherwise noted, the index begins at 1 and consists of counting numbers.

For example, a generic sequence will commonly be written as x_1, x_2, x_3… or a_1, a_2, a_3... For the sequence of even numbers given earlier, $x_1 = 2$, $x_2 = 4$, $x_3 = 6$, $x_4 = 8$, etc.

Often when we write a sequence in this form, it becomes apparent that there is a relationship between the index and the term of the sequence. In the sequence of even numbers, the term is always twice the index.

Mathematicians will use another variable, usually i or k, to represent the index, and x_i or x_k will be used to represent the i^{th} or k^{th} term in the sequence. When this is done, a formula can be written to describe the relationship between the index and the term. We will use curly brackets {} to denote that an equation represents a sequence of terms, rather than an equation we might try to solve.

For example, the sequence of even numbers can be written as $\{x_i = 2i\}$.