

$$\frac{m}{k} = \frac{+8}{+2}, \frac{+8}{-2}, \frac{-8}{+2}, \frac{-8}{-2}$$

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After removing all the repetitions, we get the following candidates as possible solutions for  $P(x) = 0$ :  $\pm 8, \pm 4, \pm 2, \pm 1, \pm \frac{1}{2}$ .

Now we only need to examine which of these 10 numbers are solutions to  $P(x) = 0$ .

We start with the easy ones:  $\pm 1$  and  $\pm 2$ .

$$2 \cdot 1^4 + 5 \cdot 1^3 + 3 \cdot 1^2 - 2 \cdot 1 - 8 = 0 \quad x = 1 \text{ is a solution}$$

$$2 \cdot (-1)^4 + 5 \cdot (-1)^3 + 3 \cdot (-1)^2 - 2 \cdot (-1) - 8 = -6 \quad x = -1 \text{ is not a solution}$$

$$2 \cdot 2^4 + 5 \cdot 2^3 + 3 \cdot 2^2 - 2 \cdot 2 - 8 = 72 \quad x = 2 \text{ is not a solution}$$

$$2 \cdot (-2)^4 + 5 \cdot (-2)^3 + 3 \cdot (-2)^2 - 2 \cdot (-2) - 8 = 0 \quad x = -2 \text{ is a solution}$$

You can check on your own that none of the other candidates is a solution to  $P(x) = 0$ . After dividing  $2x^4 + 5x^3 + 3x^2 - 2x - 8$  by  $x - 1$ , we get  $2x^3 + 7x^2 + 10x + 8$ .

Therefore, we have  $2x^4 + 5x^3 + 3x^2 - 2x - 8 = (2x^3 + 7x^2 + 10x + 8)(x - 1)$ .

Now we solve two equations:

1.  $K(x) = x - 1 = 0$
2.  $Q(x) = 2x^3 + 7x^2 + 10x + 8 = 0$

The first will give us  $x = 1$ . To solve the second equation, we need to factor  $Q(x)$ . We have:

$$P(x) = Q(x)K(x).$$

Recall  $x = -2$  solves the equation  $P(x) = 0$ . Hence,  $P(-2) = Q(-2)K(-2) = 0$ .

Since  $K(-2) \neq 0$ ,  $Q(-2) = 0$ .

Now, we use the Factor Theorem again and divide  $Q(x)$  by  $x + 2$  to get:

$$2x^3 + 7x^2 + 10x + 8 = (2x^2 + 3x + 4)(x + 2)$$

The solution of equation  $2x^3 + 7x^2 + 10x + 8 = 0$  is reduced to the solution of two equations:

1.  $x + 2 = 0$