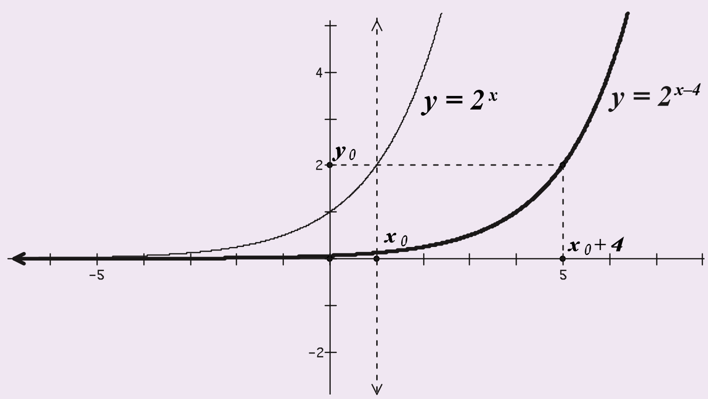
Notice that the two functions $y = f(x)$ and $y = f(x+c)$ have the same output for any two inputs that differ by c. For the inputs x and $x-c$, we have $f(x) = f((x-c)+c)$. On the basis of this fact, we can now construct the graph of $y = f(x+c)$ from the graph of $y = f(x)$.

For any point (x_0, y_0) on the graph of $y = f(x)$, we have $y_0 = f(x_0)$. What input x of the function $y = f(x+c)$ gives the output y_0? The answer is $x = x_0 - c$ because $f(x+c) = f\left((x_0 - c)+c\right) = f(x_0) = y_0$. Conversely, if $(x_0 - c, y_0)$ is on the graph of $y = f(x+c)$, then (x_0, y_0) is on the graph of $y = f(x)$.

The conclusion that we can draw from this is that to construct the graph of the function $y = f(x+c)$ from the graph of $y = f(x)$, we simply shift each point of the graph $y = f(x)$ by c units along the x-axis. We move the graph by c units to the left if $c > 0$ and by c units to the right if $c < 0$.

EXAMPLE 5.6a: Construct $y = 2^{x-4}$.

The function $y = 2^{x-4}$ is of the form $y = f(x+c)$ where $f(x) = 2^x$ and $c = -4 < 0$. Therefore, the graph of $y = 2^{x-4}$ is the result of shifting the graph of $y = 2^x$ along the x-axis to the right by 4 units.



5.6.2 Graphing y = f (x) + C from the Graph of y = f (x)

The function $y = f(x) + C$ is the composition of two functions: $y = f(x)$ and $y = x + C$. For any input x, we get the output $f(x)$. This output, in turn, becomes an input for the function $y = x + C$, giving the output $f(x) + C$.

EXAMPLES:

(1) $y = x^2 - 3$ is the composition of $y = x^2$ and $y = x - 3$: We first square an input x, and then add -3 to the result.