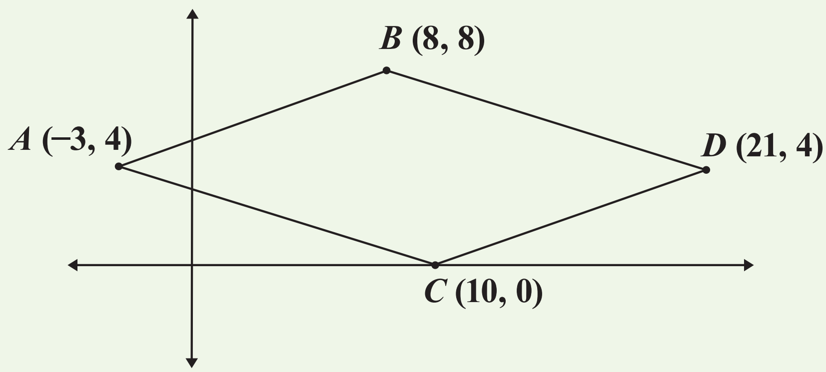

Figure 2–42

At first glance, it seems as if these side lengths are not congruent, but it looks like they may be parallel, so let's start with that. Determining the slopes of each side gives us:

Slope of AB: $\dfrac{4-8}{-3-8}=\dfrac{4}{11}$

Slope of AC: $\dfrac{4-0}{-3-10}=-\dfrac{4}{13}$

Slope of BD: $\dfrac{8-4}{8-21}=-\dfrac{4}{13}$

Slope of CD: $\dfrac{0-4}{10-21}=\dfrac{4}{11}$

Since the opposite sides of the quadrilateral are parallel, this figure is a parallelogram!

Simply knowing that this is a parallelogram, however, does not mean that we are finished with the problem. We need to make sure that there is not a more specific description that would also fit this quadrilateral. For example, if this figure was a square, it could still be classified as a parallelogram.

Ruling out square and rectangle is a good place to start since we can justify that the angles of this parallelogram are not 90°. The angles would only be 90° if the sides were perpendicular. But, as can be seen from the calculations above, the slopes of the two sides that meet at point A are not negative reciprocals, and therefore this parallelogram does not have a 90° angle at point A. The same argument applies to the other angles.

After square and rectangle have been eliminated, the only other special type of parallelogram to check for is rhombus. Although a rhombus has four congruent sides, we only need to check if two sides at one vertex are congruent. This is because a parallelogram has opposite sides that are equal in length. So, if this particular parallelogram has two adjacent sides that are congruent, then all four sides are congruent, and it will be a rhombus.

However, we can see in this case that sides AB and AC are not congruent. The slope calculations we did for it did not involve reducing fractions, and therefore the numerator of the slope of AB is the change in the y-coordinate for line segment AB ($y_1 - y_2$), and the denominator of the slope of AB is the change in the x-coordinate ($x_1 - x_2$). Therefore, these numbers can be used in the Pythagorean theorem, telling us that the length of side AB in this parallelogram is $\sqrt{137}$. Similarly, the slope calculation for AC did not require fraction reduction, so the length of side AC is $\sqrt{185}$. Therefore, AB ≠ AC, and this is just a parallelogram, not a rhombus.

Let's look at one more example and pay attention to what is necessary and what is sufficient to determine the classification of this quadrilateral.

EXAMPLE 2.2w: The vertices of a quadrilateral are H(–2,4), I(6,7), J(9,–1), and K(1,–4). What type of quadrilateral is this?

SOLUTION: As always, we start with a picture.

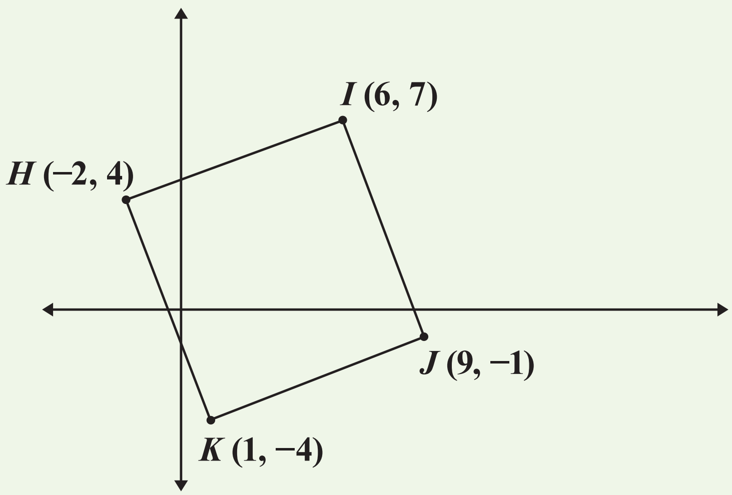

Figure 2–44

Either a rectangle or a square seems plausible. Both of these are parallelograms and have perpendicular sides. Since both of these facts rely on the slopes of the sides, calculating these slopes is once again a good place to start.

Slope HI: $\frac{4-7}{-2-6}=\frac{3}{8}$

Slope HK: $\frac{4-(-4)}{-2-1}=-\frac{8}{3}$

Slope IJ: $\frac{7-(-1)}{6-9}=-\frac{8}{3}$

Slope KJ: $\frac{-4-(-1)}{1-9}=\frac{3}{8}$

From this, we not only can determine that this quadrilateral is a parallelogram, but also can determine that the sides at each vertex are perpendicular. Therefore, this quadrilateral is either a square or a rectangle. Usually, the property cited to distinguish squares from rectangles is that a square has all four sides congruent, whereas a rectangle only has two pairs of congruent sides. The distance formula is not difficult to use, but it is annoying. Slopes, on the other hand, are much easier to calculate. As stated in the discussion of the previous problem, squares (and rhombuses and kites) have perpendicular diagonals; rectangles do not. We can distinguish between a square and a rectangle by checking the diagonals for perpendicularity. Indeed, we can see the following:

Slope HJ: $\frac{4-(-1)}{-2-9}=-\frac{5}{11}$

Slope KI: $\frac{-4-7}{1-6}=\frac{11}{5}$

Therefore, the diagonals of this parallelogram are perpendicular, and this quadrilateral is a square.

Therefore, the movement from (–5, –2) to the point closest on the line is <–3.6, 1.2>. The Pythagorean theorem says the length of this movement is $\sqrt{14.4}$.

6. As with the two-dimensional problem, the distance from the point to the plane can be found if we focus on the movement from (3, 0, 1), an arbitrary point in the plane, to (–1, 4, 6). Let (x, y, z) be the point in the plane closest to (and a perpendicular movement from) (–1, 4, 6). If $\langle a, b, c\rangle$ is the movement from (3, 0, 1) to (x, y, z) and $k\langle 3, 2, -4\rangle$ is the movement from (x, y, z) to (–1, 4, 6), we can represent this with a vector equation.

   $(3, 0, 1) + \langle a, b, c\rangle + k\langle 3, 2, -4\rangle = (-1, 4, 6)$.

   This can be split into three equations, one for each coordinate.

   x: $3 + a + 3k = -1$, or $a + 3k = -4$
   y: $0 + b + 2k = 4$, or $b + 2k = 4$
   z: $1 + c - 4k = 6$, or $c - 4k = 5$.

   Using the fact that $\langle a, b, c\rangle$ and $\langle 3, 2, -4\rangle$ are perpendicular, we multiply the first equation by 3, the second equation by 2, and the third equation by –4, and adding together yields:

   $3a + 9k + 2b + 4k - 4c + 16k = -24$, so $29k = -24$, and $k = \frac{-24}{29}$.

   Therefore, the movement from (x, y, z) to the point (–1, 4, 6) is $\frac{-24}{29}\langle 3, 2, -4\rangle$, and the Pythagorean theorem gives the length of this distance as $\frac{24}{29} \cdot \sqrt{29}$.

7. Let a be the first x-coordinate; this means that (a, a^2) is the first point on the graph. Since the two x-coordinates sum to 5, this means that $5 - a$ is the second x-coordinate, which means $(5 - a, (5 - a)^2)$ is the second point on the graph. We want to show that the slope between these two points is 5.

   $$\frac{a^2 - (5 - a)^2}{a - (5 - a)} = \frac{a^2 - 25 + 10a - a^2}{2a - 5} = \frac{5(2a - 5)}{2a - 5} = 5.$$

8. All the points that are a distance of 5 away from the origin would describe a sphere of radius 5. The equation for this sphere can be created from the distance formula in three dimensions. If (x, y, z) is any point on the sphere, the distance from (x, y, z) to the origin is 5. Therefore, $\sqrt{(x - 0)^2 + (y - 0)^2 + (z - 0)^2} = 5$, or simplifying, $x^2 + y^2 + z^2 = 25$.

9. As usual, we start by drawing a picture.

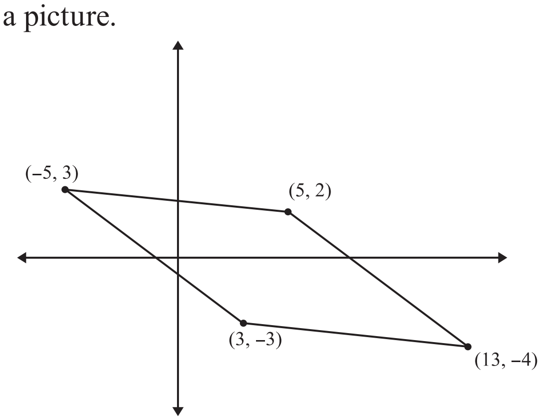

Figure 2–105